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ASYMPTOTIC PERFORMANCE OF A SYSTEM SUBJECT TO CANNIBALIZATION

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ABSTRACT

We consider the performance of a system composed of n machines, each machine being composed of parts (or modules) of r different types arranged in series. If S_i denotes the number of parts of type i functioning at a given moment, it is assumed that the performance level, Φ, of the overall system at that moment is given by the equation

$$\Phi = \min_{1 \leq i \leq r} S_i \,.$$

A collection of identical structures (ships, aircraft, missile batteries, etc.) exemplifies the kind of system we have in mind, under the simplifying assumption that each part in a given structure is essential to the performance of that structure. Limit theorems are proved which make it possible to evaluate explicitly the asymptotic performance level of the system under three different assumptions:

1. Cannibalization is prohibited.
2. A failed part in a given machine can be replaced by an operating part of the same type taken from another machine. The lifetimes of the parts are mutually independent random variables.
3. A failed part in a given machine can be replaced by an operating part of the same type taken from another machine. Failures of parts may affect the lifetimes of other parts.

It is shown that assumptions (2) and (3) lead to asymptotically equivalent performance levels. The superiority of systems in which cannibalization is practiced over those in which it is prohibited is quantitatively appraised.

INTRODUCTION

Among possible ways of maintaining a system[1] whose spares have been exhausted, cannibalization is one that has been practiced over a long period of time by the military of many nations, albeit not always with the consent of the commanding officer. By "cannibalization" we mean the removal of a failed part[2] from a system and its replacement by an operating part of the same type extracted from another part of the same system. In a system consisting of two aircraft, a simple example of cannibalization is the switching of a damaged tire on one aircraft with a good tire on the second. If, for instance, the radar on the second aircraft is damaged and a replacement is unavailable, the exchange of tires may at least restore one aircraft to operating condition.

The practice of cannibalization is highly controversial primarily because the very act of extracting an operating part from a complex system can cause extensive damage to other parts of the system. On the other hand, in situations where it is crucial to maintain the performance of a system at a high level, a viable alternative to cannibalization may be difficult to find. Because of the importance and polemical nature of cannibalization,

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[1]The word "system" is not limited to a single physical entity such as an aircraft, ship, missile launcher, etc. It includes also any designated collection of objects whose maintenance and performance as a unit is of interest.

[2]The failed part can be either a single part or an entire module, i.e., a package of components of a system that can be removed and replaced as a whole. Throughout this paper the word "part" can be interpreted to mean "individual part" or "module".

[illegible]

[illegible] probability distribution and expected value of the operating performance levels. In the present paper we [illegible] further the [illegible] in [illegible] in detail a special class of systems whose parts have [illegible] exponential failure rates. In this special case, as will be shown, it is possible to derive limit theorems that make it computationally feasible to assess the effects of cannibalization and to compare the performance levels and efficiencies of cannibalized and uncannibalized systems.

I. DESCRIPTION OF THE SYSTEM

We consider a collection of n objects, O_1, O_2,..., O_n, to be used as a system to carry out a sequence of specified tasks, each task requiring τ units of time to complete. For simplicity we assume that the k-th task is initiated at time $(k-1)\tau$ and completed at time $k\tau$, $k = 1,2,\ldots$.[3] We shall refer to each object as a "machine" and to each task as a "mission". We define the size of the system to be the number, n, of objects it contains. A typical example of the kind of system we have in mind is a fleet of destroyers sent on voyages, or a squadron of aircraft sent on reconnaissance missions. We emphasize, however, that the nature of the machines and the kinds of missions performed may be quite arbitrary. Each machine O_j is composed of r distinct parts, $\gamma_{1j},\gamma_{2j},\ldots,\gamma_{rj}$, $j = 1,2,\ldots,n$, where $r > 1$ and for each index i, $1 \leq i \leq r$, the parts $\gamma_{i1},\gamma_{i2},\ldots,\gamma_{in}$ are assumed to be replicas of each other.

A machine is sent on a mission if and only if all of its parts are in good working order at the beginning of the mission. If one or more of its parts fails during the course of a mission, it is assumed that the machine returns from the mission, i.e., that the machine and its unfailed parts are available at the end of the mission.[4] We consider the case when no spare parts are

[3]Only minor modifications are needed to treat the case when the tasks are carried out during arbitrarily specified non-overlapping intervals of time of length τ.

[4]This assumption is not as pernicious as it may seem, since the machines being considered may be subassemblies of larger pieces of equipment. For example, the machines may be periscopes in submarines; even if the periscope is damaged during the course of a mission, the submarine has the capacity to return to its base.

[illegible] and unused parts are irreparable, so that cannibalization is the only possible mode of maintenance. We assume that for each index i, $1 \leq i \leq r$, the parts $p_{i1}, p_{i2}, \ldots, p_{in}$ are freely interchangeable, and that these are the only allowed exchanges. At the conclusion of a given mission k, the system is cannibalized in such a way as to maximize the number of machines that can be dispatched on mission k+1. The assumption that missions are dispatched at times $0, \tau, 2\tau, \ldots$ and are of duration τ implies that all cannibalizations are instantaneously carried out.[5]

It is convenient to describe the state of each part at a given moment t by means of an ordered pair (x,y) whose first component x indicates the "employment status" of the part at time t and whose second component y indicates the "fitness" of the part at time t. For our analysis it suffices to distinguish two categories of employment: If, at time t, the part is installed in a machine that is then on a mission, we set x = 1 and say that the part is "engaged"; if it is in a machine that is not on a mission at time t, we set x = 0 and say that the part is "idle."[6] Concerning fitness, we assume that each part is either in good working order (operational) or has failed (unfit). We set y = 1 if the part is in good working order at time t; otherwise, y = 0. Thus, at each instant the state of a given part corresponds to one of the four vertices (0,0), (0,1), (1,0), (1,1) of the unit

[5] This restriction can easily be removed without essential modification of our conclusions.

[6] We adopt the convention that the instants $0, \tau, 2\tau, \ldots$ when the missions are initiated are engaged times for parts in the dispatched machines.

square. It is assumed that all parts are operational at time 0.

We shall analyze the performance of the system under two distinct assumptions concerning the periods when a part is exposed to risk of failure. The first of these, called the assumption of "synchronous clocks," postulates that at every instant t each part which is operational at time t, regardless of its employment status, is subject to risk of failure. This assumption (adopted in [1]) contrasts with the assumption of "asynchronous clocks" (adopted in [2]) according to which a given part is subject to risk of failure at time t if and only if it is in state (1,1) at time t, i.e., engaged and operational.[7] In both cases – synchronous and asynchronous clocks – it will be assumed that when parts are simultaneously exposed to risk, they fail independently of each other. Moreover, the failure rate of a part at any given instant when it is exposed to risk does not depend on the particular machine in which the part is installed nor on the sequence of machines through which it has passed.

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[7]The terms "synchronous clocks" and "asynchronous clocks" are motivated by the following interpretation: We imagine that each part is equipped with its own clock, all clocks being set to 0 and turned on at time 0. Under the assumption of synchronous clocks, no clock is ever turned off. Under the assumption of asynchronous clocks, the clock associated with a given part is turned off when the part is not engaged; hence, under this assumption two operational parts, one of which is in state (0,1) during the time interval $[(k-1)\tau, k\tau]$ and the other of which is in state (1,1) during this time interval, will, at time $k\tau$, have clocks showing different times if at time $(k-1)\tau$ they showed the same time.

[illegible]

In this section we restrict attention to the case of [illegible]. Suppose that at the time $k\tau$, when the [illegible] is inspected, there are N_i operational parts of type i, i = 1,2,...,r. Setting

$$\Phi = \min_{1 \leq i \leq r} N_i ,$$

it is clear that by appropriate cannibalizations exactly Φ machines can be assembled, all of whose parts are operational. It therefore seems reasonable to define Φ to be the "performance level" of the system at time $k\tau$.

We now associate to each part γ_{ij} a random variable X_{ij}, called the "lifetime" of γ_{ij}, whose value denotes the time of failure of γ_{ij}. We shall assume that each random variable X_{ij} is exponentially distributed with distribution function

$$P\{X_{ij} < t\} = \begin{cases} 1 - e^{-\alpha_i t} , & t > 0 , \\ 0 , & t \leq 0 . \end{cases}$$

We emphasize that the failure rate α_i of γ_{ij} depends only on the part type i. Clearly the expected lifetime of a part of type i is

$$E[X_{ij}] = \alpha_i^{-1} > 0 .$$

Under the assumption of independent failures (stated in Section 1) it follows that the random variables $(X_{ij})_{\substack{i=1,2,\ldots,r \\ j=1,2,\ldots,n}}$ are independent.

We obtain a convenient expression for the number of operational parts of each type at time $k\tau$ by introducing the indicators $Y_{ij}(k)$ of the events $\{X_{ij} \geq k\tau\}$, defined for $k \geq 0$ and the admissible values of i,j by the equations

$$Y_{ij}(k) = \begin{cases} 1 \ , & \text{if } X_{ij} \geq k\tau \ , \\ 0 \ , & \text{otherwise.} \end{cases}$$

Plainly, for each index k the random variables $(Y_{ij}(k))_{\substack{i=1,2,\ldots,r \\ j=1,2,\ldots,n}}$ are independent, and

$$(1) \qquad Y_{ij}(k) = \begin{cases} 1 \ , & \text{with probability } e^{-\alpha_i k\tau} \ , \\ 0 \ , & \text{with probability } 1 - e^{-\alpha_i k\tau} \ . \end{cases}$$

Let

$$(2) \qquad S_{in}(k) = \sum_{j=1}^{n} Y_{ij}(k) \ .$$

Then $S_{in}(k)$ denotes the number of operational parts of type i at time $k\tau$, given that the size of the system is n. Setting

$$p_i(k) = e^{-\alpha_i k\tau}$$

and

$$q_i(k) = 1 - p_i(k) \ ,$$

it follows from (1) and (2) and the independence of $(Y_{ij}(k))_{j=1,2,\ldots,n}$ that

$$(3) \qquad P\{S_{in}(k) = m\} = \binom{n}{m} p_i^m(k) q_i^{n-m}(k) \ , \quad m = 0,1,\ldots,n \ .$$

By definition, the system performance level at time $k\tau$ is given by the random variable

$$\Phi_n(k) = \min_{1 \leq i \leq r} S_{in}(k)$$

We observe now that for each pair of indices (n,k) the random variables $S_{1n}(k), S_{2n}(k), \ldots, S_{rn}(k)$ are independent, since the random vectors

$$(Y_{11}(k), Y_{12}(k), \ldots, Y_{1n}(k)),\ (Y_{21}(k), Y_{22}(k), \ldots, Y_{2n}(k)),\ \ldots,\ (Y_{r1}(k), Y_{r2}(k), \ldots, Y_{rn}(k))$$

are independent. Hence, the expected performance level at time $k\tau$ can be determined from the tail distribution by the well-known formula,

$$(4)\quad E[\Phi_n(k)] = \sum_{\lambda=0}^{n-1} P\{\Phi_n(k) > \lambda\}$$

$$= \sum_{\lambda=0}^{n-1} P\{S_{1n}(k) > \lambda,\ S_{2n}(k) > \lambda,\ \ldots,\ S_{rn}(k) > \lambda\}$$

$$= \sum_{\lambda=0}^{n-1} \prod_{i=1}^{r} P\{S_{in}(k) > \lambda\} = \sum_{\lambda=0}^{n-1} \prod_{i=1}^{r} \sum_{m=\lambda+1}^{n} \binom{n}{m} p_i^m(k) q_i^{n-m}(k)$$

$$= \sum_{\lambda=1}^{n} \prod_{i=1}^{r} \sum_{m=\lambda}^{n} \binom{n}{m} p_i^m(k) q_i^{n-m}(k)\ .$$ [8]

It follows immediately from the definition of $\Phi_n(k)$ that the expected system performance level is, for fixed k, an increasing function of n, i.e.,

$$E[\Phi_{n+1}(k)] \geq E[\Phi_n(k)]\ , \quad n = 1,2,\ldots\ .$$

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[8] This formula is a special case of equations (4.3) and (4.4) in [1], which give the expected performance level of an arbitrary system subject to cannibalization.

Enlarging the size of the system, however, entails added cost, and it is therefore of interest to introduce some measure of the return per dollar of investment. To this end we define the "efficiency" $\rho_n(k)$ of a system after k missions, given that the system has size n, as the expected performance per machine; thus,

$$\rho_n(k) = \frac{1}{n} E[\Phi_n(k)] . \tag{5}$$

We emphasize that, in general, the efficiency depends not only on the time $k\tau$ at which it is evaluated but also on the size, n, of the system. It is of interest to study the behavior of $\rho_n(k)$ as n varies and to compare the efficiencies of cannibalized and uncannabilized systems.

Let us consider the system described in Section 1, assuming that cannibalizations are not performed. Let $\Phi_n^o(k)$ and $\rho_n^o(k)$ denote, respectively, the performance level and efficiency after k missions, given that the system has size n. Since a given machine will be operational at time $k\tau$ if and only if all of its parts survive k missions, and this occurs with probability $p_1(k)p_2(k)\cdots p_r(k)$, it is clear that

$$P\{\Phi_n^o(k)=m\} = \binom{n}{m}[p_1(k)p_2(k)\cdots p_r(k)]^m[1-p_1(k)p_2(k)\cdots p_r(k)]^{n-m}.$$

Hence, by an elementary formula,

$$E[\Phi_n^o(k)] = np_1(k)p_2(k) \cdots p_r(k) ,$$

and

$$(6) \qquad P_n(k) = p_1(k)\,p_2(k) \cdots p_r(k) \; .$$

From (4) and (5) we see that the efficiency of the system when cannibalization is practiced is given by the expression

$$(7) \qquad \rho_n(k) = \frac{1}{n} \sum_{\lambda=1}^{n} \prod_{i=1}^{r} \sum_{m=\lambda}^{n} \binom{n}{m} p_i^m(k)\, q_i^{n-m}(k) \; .$$

We note from (6) that the efficiency of the uncannibalized system is a constant depending only on the part failure rates and is independent of the system size, whereas from (7) it is clear that the efficiency of the cannibalized system varies with n, although it is by no means obvious precisely how the efficiency changes as the system is enlarged.

The tables in the Appendix show how the efficiencies of the cannibalized and uncannibalized systems compare in various special cases. These tables suggest that for each fixed time $k\tau$ the efficiency of the cannibalized system is a strictly increasing function of the system size. For simplicity we present here a proof of this result only in a very special case, leaving for another paper a discussion of the full theorem.

THEOREM 1. Let $\rho_n(k)$ denote the efficiency after k missions of the system described in Section 1, assuming that cannibalization is practiced, each machine is composed of two parts, the failure rates of the parts are equal, and clocks are synchronous. Then for each positive integer k,

$$\rho_n(k) < \rho_{n+1}(k) \; , \qquad n = 1,2,\ldots \; .$$

Proof. Since k will be held fixed throughout our argument, for simplicity we suppress k in all our notation. From the definition of efficiency,

(8) $$\rho_{n+1} - \rho_n = E\left[\frac{\Phi_{n+1}}{n+1} - \frac{\Phi_n}{n}\right] = \frac{1}{n(n+1)} E[n(\Phi_{n+1} - \Phi_n) - \Phi_n] .$$

Setting

$$\Phi_0 = 0$$

and

$$\Delta_n = \Phi_{n+1} - \Phi_n , \qquad n = 0,1,\ldots,$$

from (8) we see that it suffices to prove

(9) $$E[n\Delta_n - \Phi_n] > 0 , \qquad n = 1,2,\ldots .$$

Writing Φ_n in the form

$$\Phi_n = \sum_{j=0}^{n-1} \Delta_j , \qquad n = 1,2,\ldots,$$

inequality (9) reduces to

(10) $$E[n\Delta_n] - \sum_{j=0}^{n-1} E[\Delta_j] > 0 , \quad n=1,2,\ldots .$$

We define for $n \geq 1$,

(11) $$u_n = P\{S_{1n} = S_{2n}\} .$$

It is convenient to introduce the additional definition,

$$u_0 = 1 .$$

Setting

$$p_1(k) = p_2(k) = p ,$$

It is easy to verify that for $n \geq 1$, Δ_n takes only the values [illegible], with, respectively, the probabilities

$$P\{\Delta_n = 1\} = u_n p^2 + (1-u_n)p ,$$

$$P\{\Delta_n = 0\} = 1 - [u_n p^2 + (1-u_n)p] .$$

Since $E\{\Delta_n\} = P\{\Delta_n = 1\}$, ([illegible]) is equivalent to the inequality

$$n[u_n p^2 + (1-u_n)p] - \sum_{j=0}^{n-1} [u_j p^2 + (1-u_j)p] > 0 ,$$

which in turn reduces to

$$(12) \qquad p(1-p)(nu_n - \sum_{j=0}^{n-1} u_j) < 0 .$$

By Abel's partial summation formula,

$$nu_n - \sum_{j=0}^{n-1} u_j = \sum_{j=1}^{n} j(u_j - u_{j-1}) .$$

Since $p(1-p) > 0$, to prove (12) it suffices to establish the inequality

$$(13) \qquad \sum_{j=1}^{n} j(u_j - u_{j-1}) < 0 .$$

We shall in fact prove the stronger result that for all positive integers j,

$$(14) \qquad u_j < u_{j-1} .$$

To this end we note from (3), (11), and the independence of the random variables S_{1n}, S_{2n} that

$$u_n = \sum_{m=0}^{n} \binom{n}{m}^2 p^{2m} (1-p)^{2(n-m)} , \quad n = 0,1,\ldots .$$

Let us recall that if f is the polynomial

$$f(z) = \sum_{m=0}^{j} a_m z^m ,$$

then

$$(15) \qquad \frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})\bar{f}(e^{i\theta})\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} |f(e^{i\theta})|^2\, d\theta = \sum_{m=0}^{j} |a_m|^2 .$$

Setting $q = 1-p$ and applying (15) to the particular function

$$f(z) = (p+qz)^j = \sum_{m=0}^{j} \binom{j}{m} p^m q^{j-m} z^{j-m} ,$$

we obtain

$$(16) \qquad \frac{1}{2\pi}\int_0^{2\pi} |p+qe^{i\theta}|^{2j}\, d\theta = \sum_{m=0}^{j} \binom{j}{m}^2 p^{2m} q^{2(j-m)} = u_j .$$

Now noting that for $0 < \theta < 2\pi$,

$$|p+qe^{i\theta}|^2 < p+q = 1 ,$$

and for $\theta = 0$ or 2π,

$$|p+qe^{i\theta}|^2 = 1 ,$$

(14) follows from (16). This completes the proof of the theorem.

[illegible]

$$\mu_n(k) = \frac{[illegible]}{[illegible]} .$$

The limit

$$\mu_\infty(k) = \lim_{n\to\infty} \mu_n(k) \quad , \tag{17}$$

if it exists, can be interpreted as a measure of the advantage of cannibalizing a very large system in the time interval $[0,k\tau]$. We call $\mu_\infty(k)$ the "maintenance index for k missions." The limit

$$\mu^* = \lim_{k\to\infty} \mu_\infty(k) \quad ,$$

if it exists, is called the "steady state maintenance index"; it provides an estimate of the long-run superiority of a very large cannibalized system over an uncannibalized one.

THEOREM 2. In the case of synchronous clocks, the maintenance indices of the system described in Section 1 satisfy the equations

$$\mu_\infty(k) = \frac{\min\limits_{1\le i\le r} p_i(k)}{p_1(k)p_2(k)\cdots p_r(k)} \tag{18}$$

and

$$\mu^* = \infty \quad . \tag{19}$$

Proof. By the Strong Law of Large Numbers,

$$\lim_{n\to\infty} \frac{1}{n} S_{in}(k) = p_i(k) \quad \text{a.s.}, \qquad i = 1,2,\ldots,r.$$

Consequently,

$$\lim_{n\to\infty} \frac{\Phi_n(k)}{n} = \lim_{n\to\infty} \min_{1\le i\le r} \frac{1}{n} S_{in}(k) = \min_{1\le i\le r} p_i(k) \quad \text{a.s.}$$

Since $|\frac{1}{n} \Phi_n(k)| \le 1$, by the Lebesgue bounded convergence theorem it follows that

$$\lim_{n\to\infty} E[\frac{1}{n} \Phi_n(k)] = \min_{1\le i\le r} p_i(k) .$$

Hence, using (6), we have

$$\mu_\infty(k) = \frac{\min\limits_{1\le i\le r} p_i(k)}{p_1(k)p_2(k)\cdots p_r(k)} ,$$

which, from the definition of $p_i(k)$, reduces to

$$\mu_\infty(k) = \left(\frac{\min\limits_{1\le i\le r} e^{-\alpha_i\tau}}{e^{-\sum_{i=1}^{r} \alpha_i\tau}} \right)^k .$$

Since

$$\frac{\min\limits_{1\le i\le r} e^{-\alpha_i\tau}}{e^{-\sum_{i=1}^{r} \alpha_i\tau}} = \min_{1\le i\le r} e^{-\alpha_i\tau} e^{\sum_{i=1}^{r} \alpha_i\tau} > 1 ,$$

we have

$$\lim_{k\to\infty} \mu_\infty(k) = \infty ,$$

which completes the proof.

[illegible]

[illegible]

[illegible] $i=1,2,\dots,r$; $j=1,2,\dots,n$ [illegible] independent random variables. We confine ourselves in this case to showing that the maintenance indices are precisely the same as they are in the case dealt with in the previous section.

THEOREM [illegible]. In the case of asynchronous clocks, the maintenance indices of the system described in Section 1 satisfy the equations

$$\mu_\alpha(k) = \frac{\min\limits_{1\le i\le r} p_i(k)}{p_1(k)p_2(k)\cdots p_r(k)}$$

and

$$\mu^* = \infty \ .$$

Proof. Let $(X_{ij}(k))_{\substack{i=1,2,\dots,r\\ j=1,2,\dots,n\\ k=1,2,\dots}}$ be a family of random variables such that for each k the random variables $(X_{ij}(k))_{\substack{i=1,2,\dots,r\\ j=1,2,\dots,n}}$ are independent, and for all i,j,k,

$$X_{ij}(k) = \begin{cases} 1 \ , & \text{with probability } p_i(1) \ , \\ 0 \ , & \text{with probability } 1-p_i(1) \ . \end{cases}$$

The random variable $X_{ij}(1)$ corresponds to a part of type i used on the first mission; $X_{ij}(1) = 1$ if and only if the part to which

it corresponds survives the first mission. Thus, the random variable $\sum_{j=1}^{n} X_{ij}(1)$ denotes the number of parts of type i operational at the outset of the second mission, and the random variable

$$(20) \qquad N_1 = N_1(n) = \min_{1 \le i \le r} \sum_{j=1}^{n} X_{ij}(1)$$

denotes the number of operational machines obtained by cannibalization after the first mission. For each fixed index i the random variables $X_{ij}(2)$, $j = 1,2,\ldots,N_1$, correspond to the N_1 parts of type i used on the second mission; $X_{ij}(2) = 1$ if and only if the part to which it corresponds survives the second mission. The assumed independence of the random variables $(X_{ij}(2))_{\substack{i=1,2,\ldots,r \\ j=1,2,\ldots,n}}$ reflects the fact that when parts are simultaneously exposed to risk, they fail independently of each other. Clearly the number of operational parts of type i available after the second mission consists of parts of type i dispatched on the second mission that survive the second mission, i.e., $\sum_{j=1}^{N_1} X_{ij}(2)$, plus parts of type i that survived the first mission but were not used on the second, i.e., $\sum_{j=1}^{n} X_{ij}(1) - N_1$. Thus, the number of operational machines obtained by cannibalization after the second mission is given by the random variable

$$(21) \quad N_2 = N_2(n) = \min_{1 \le i \le r} \left[\sum_{j=1}^{n} X_{ij}(1) + \sum_{j=1}^{N_1} X_{ij}(2) - N_1 \right] .$$

More generally, we define recursively a sequence of random variables

$$N_1, N_2, \ldots$$

by choosing N_1 according to (20); then, assuming $N_1, N_2, \ldots, N_{k-1}$

[illegible] been defined, [illegible]

$$N_k = N_k(n) = \min_{1\le i\le r} \Big[\sum_{j=1}^{n} X_{ij} [\text{illegible}] + \sum [\text{illegible}] + \dots + \sum_{j=1}^{n-1} X_{ij}(k) - \sum_{j=1}^{k-1} [\text{illegible}] \Big] .$$

By an argument similar to the one used in deriving [illegible] it follows that N_k is the number of operational machines attained by cannibalization after the k-th mission, i.e., it is the system performance level at time $k\tau$.

To obtain the maintenance index for k missions we shall prove inductively that for all integers k,

$$\lim_{n\to\infty} \frac{1}{n} N_k(n) = \min_{1\le i\le r} p_i(k) .$$

To shorten notation we set

$$p_i = p_i(1) = e^{-\alpha_i \tau}$$

and

$$\delta = \min_{1\le i\le r} p_i > 0 .$$

By the Strong Law of Large Numbers,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} X_{ij}(1) = p_i \quad \text{a.s.}$$

Hence,

(24) $$\lim_{n\to\infty} \frac{1}{n} N_1(n) = \delta \quad \text{a.s.},$$

i.e.,

$$N_1(n) \sim n\delta \quad \text{a.s.}$$

Assume that for $\nu \leq k-1$,

$$N_\nu(n) \sim n\delta^\nu \quad , \text{ a.s.}$$

Since this implies that $N_\nu(n) \to \infty$ as $n \to \infty$, we obtain from the Strong Law of Large Numbers the relation

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{N_\nu} X_{ij}(\nu+1) = \lim_{n\to\infty} \frac{N_\nu}{n} \frac{1}{N_\nu} \sum_{j=1}^{N_\nu} X_{ij}(\nu+1) = \delta^\nu p_i \quad , \text{ a.s.}$$

Hence, we conclude from (22) that a.s.

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{n} N_k(n) &= \min_{1\leq i\leq r} \left(p_i + \delta p_i + \ldots + \delta^{k-1} p_i - \sum_{\nu=1}^{k-1} \delta^\nu\right) \\
&= \min_{1\leq i\leq r} \left(p_i \sum_{j=0}^{k-1} \delta^j - \sum_{\nu=1}^{k-1} \delta^\nu\right) \\
&= \left(\sum_{j=0}^{k-1} \delta^j\right)\left(\min_{1\leq i\leq r} p_i\right) - \sum_{\nu=1}^{k-1} \delta^\nu \\
&= \sum_{j=1}^{k} \delta^j - \sum_{\nu=1}^{k-1} \delta^\nu = \delta^k \ .
\end{aligned}$$

This completes the induction. Since

$$\delta^k = \left(\min_{1\leq i\leq r} p_i(1)\right)^k = \left(\min_{1\leq i\leq r} e^{-\alpha_i \tau}\right)^k = \min_{1\leq i\leq r} e^{-\alpha_i k\tau} = \min_{1\leq i\leq r} p_i(k),$$

(23) is proved. From the definition of N_k it is obvious that $\left|\frac{N_k}{n}\right| \leq 1$, so that by the Lebesgue bounded convergence theorem,

Plainly, the efficiency of the [illegible] maintained system after a mission does not depend on whether or not clocks are synchronous. It follows that the maintenance indices are the same as in the [illegible].

Acknowledgment: We should like to acknowledge that our interest in the subject matter of this paper was stimulated by results of A. J. Rolfe [[illegible]], whose numerical computations suggested the validity of the theorems we have proved. We are indebted also to Professor Agnes Berger for many stimulating discussions in connection with Theorem 1.

APPENDIX

TABLE 1

Efficiency After k Missions of a System with 2 Part Types

System Sizes, 2-13

$p_1(1) = .9,\ p_2(1) = .95$

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	2	3	4	5	6	7	8	9	10	11	12	13	
1	.859	.863	.866	.869	.871	.873	.875	.877	.878	.880	.881	.882	.855
2	.745	.755	.763	.769	.774	.778	.781	.784	.786	.788	.790	.791	.731
3	.649	.665	.676	.685	.691	.696	.700	.703	.706	.708	.710	.712	.625
4	.568	.589	.602	.612	.619	.624	.628	.632	.634	.637	.639	.640	.534
5	.499	.523	.537	.547	.554	.560	.564	.568	.570	.573	.575	.576	.457
6	.439	.464	.479	.490	.497	.502	.507	.510	.513	.515	.517	.519	.391
7	.387	.412	.428	.438	.446	.451	.455	.459	.461	.464	.465	.467	.334
8	.340	.366	.382	.392	.399	.405	.409	.412	.415	.417	.419	.420	.286
9	.299	.326	.341	.351	.358	.363	.367	.370	.373	.375	.377	.378	.245
10	.263	.289	.304	.314	.321	.326	.330	.333	.335	.337	.339	.340	.209
11	.231	.256	.271	.281	.287	.292	.296	.299	.301	.303	.305	.306	.179
12	.203	.227	.241	.251	.257	.262	.266	.269	.271	.273	.274	.275	.153
13	.178	.201	.215	.224	.230	.235	.238	.241	.243	.245	.246	.247	.131
14	.156	.178	.191	.200	.206	.210	.214	.216	.218	.220	.221	.222	.112
15	.136	.157	.170	.178	.184	.188	.192	.194	.196	.198	.199	.200	.096

[illegible]

[illegible]

System Sizes, 14-[illegible]

[illegible]

Vis. K	[illegible] System of Size												[illegible] System of Any Size
	[illegible]	[illegible]	16	17	1[illegible]	19	20	21	[illegible]	[illegible]	[illegible]	[illegible]	
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.887	.888	.888	.889	[illegible]	.890	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	.797	.798	.798	.799	[illegible]	.800	.801	.801	.731
3	.713	.715	.716	.717	.718	.718	.719	.720	.720	.721	.722	.722	.625
4	.642	.643	.644	.645	.646	.647	.648	.648	.649	.650	.650	.651	.534
5	.578	.579	.580	.581	.582	.583	.584	.584	.585	.585	.586	.586	.457
6	.520	.521	.523	.523	.524	.525	.526	.526	.527	.527	.527	.528	.391
7	.468	.469	.470	.471	.472	.473	.473	.474	.474	.475	.475	.475	.334
8	.422	.423	.423	.424	.425	.426	.426	.427	.427	.427	.428	.428	.286
9	.379	.380	.381	.382	.382	.383	.384	.384	.384	.385	.385	.385	.245
10	.341	.342	.343	.344	.344	.345	.345	.346	.346	.346	.347	.347	.209
11	.307	.308	.309	[illegible]	.310	.310	.311	.311	.311	.312	.312	.312	.179
12	.276	.277	.278	.278	.279	.279	.280	.280	.280	.281	.281	.281	.153
13	.248	.249	.250	.250	.251	.251	.252	.252	.252	.252	.253	.253	.131
14	.223	.224	.225	.225	.226	.226	.226	.227	.227	.227	.227	.228	.113
15	.201	.202	.202	.203	.203	.203	.204	.204	.204	.204	.205	.205	.096

TABLE 3

Efficiency after k Missions of a System with 3 Part Types

System Sizes, 2-13

$p_1(1) = .9,\ p_2(1) = .92,\ p_3(1) = .96$

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	2	3	4	5	6	7	8	9	10	11	12	13	
1	.807	.817	.825	.832	.837	.842	.846	.850	.853	.856	.858	.860	.795
2	.667	.691	.709	.721	.731	.738	.744	.749	.754	.757	.761	.763	.632
3	.561	.595	.617	.632	.643	.652	.659	.665	.670	.674	.677	.680	.502
4	.476	.515	.540	.556	.568	.577	.585	.591	.596	.600	.604	.607	.399
5	.406	.448	.473	.490	.502	.512	.519	.525	.531	.535	.539	.542	.317
6	.346	.389	.414	.431	.444	.454	.461	.467	.473	.477	.481	.484	.252
7	.295	.337	.363	.380	.392	.402	.410	.416	.421	.425	.429	.432	.200
8	.250	.292	.317	.334	.347	.356	.364	.370	.375	.379	.383	.386	.159
9	.212	.253	.277	.294	.306	.315	.323	.329	.334	.338	.341	.344	.126
10	.179	.218	.242	.258	.270	.279	.286	.292	.297	.301	.304	.307	.100
11	.150	.188	.211	.226	.238	.247	.253	.259	.264	.268	.271	.274	.080
12	.126	.161	.183	.198	.209	.218	.224	.230	.234	.238	.241	.244	.064
13	.105	.138	.159	.173	.184	.192	.198	.203	.208	.211	.215	.217	.051
14	.087	.118	.137	.151	.161	.169	.175	.180	.184	.188	.191	.193	.041
15	.072	.100	.118	.131	.141	.148	.154	.159	.163	.166	.169	.172	.033

TABLE 4

Efficiency After k Missions of a System with 3 Part Types

System Sizes, 14–25

$p_1(1)$ = [illegible], $p_2(1)$ = [illegible], $p_3(1)$ = [illegible]

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	14	15	16	17	18	19	20	21	22	23	24	25	
1	[illegible]	.864	.865	.867	.868	.869	.870	.871	.872	.873	.874	.875	.795
2	.766	.768	.770	.772	.773	.775	.776	.778	.779	.780	.781	.782	.632
3	.683	.685	.688	.690	.691	.693	.695	.696	.697	.698	.699	.700	.502
4	.610	.613	.615	.617	.619	.620	.622	.623	.625	.626	.627	.628	.399
5	.545	.548	.550	.552	.554	.556	.557	.559	.560	.561	.562	.563	.317
6	.487	.490	.492	.494	.496	.498	.499	.500	.502	.503	.504	.505	.252
7	.435	.438	.440	.442	.444	.446	.447	.448	.450	.451	.452	.453	.200
8	.389	.391	.393	.395	.397	.399	.400	.402	.403	.404	.405	.406	.159
9	.347	.350	.352	.354	.355	.357	.358	.360	.361	.362	.363	.364	.126
10	.310	.312	.314	.316	.318	.319	.321	.322	.323	.324	.325	.326	.100
11	.276	.279	.281	.283	.284	.286	.287	.288	.289	.291	.292	.292	.080
12	.246	.249	.251	.252	.254	.255	.257	.258	.259	.260	.261	.262	.064
13	.220	.222	.224	.225	.227	.228	.229	.231	.232	.233	.234	.234	.051
14	.196	.198	.199	.201	.202	.204	.205	.206	.207	.208	.209	.210	.041
15	.174	.176	.178	.179	.181	.182	.183	.184	.185	.186	.187	.188	.033

TABLE 5

Efficiency After k Missions of a System with 4 Part Types

System Sizes, 2-13

$p_1(1) = .9,\ p_2(1) = .92,\ p_3(1) = .95,\ p_4(1) = .97$

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	2	3	4	5	6	7	8	9	10	11	12	13	
1	.781	.796	.808	.817	.825	.831	.837	.841	.845	.849	.852	.854	.763
2	.634	.668	.690	.706	.718	.728	.735	.741	.746	.751	.754	.758	.582
3	.527	.572	.600	.618	.632	.642	.650	.657	.663	.668	.672	.675	.444
4	.444	.494	.523	.543	.557	.568	.577	.584	.590	.595	.599	.603	.339
5	.375	.427	.457	.477	.492	.503	.512	.519	.525	.530	.534	.538	.259
6	.318	.369	.399	.419	.434	.445	.454	.461	.467	.473	.477	.481	.198
7	.268	.318	.348	.368	.383	.394	.403	.410	.416	.421	.425	.429	.151
8	.224	.274	.304	.324	.338	.349	.358	.365	.370	.375	.379	.383	.115
9	.187	.235	.264	.284	.298	.309	.317	.324	.329	.334	.338	.342	.088
10	.155	.202	.230	.249	.262	.273	.281	.287	.293	.297	.301	.305	.067
11	.128	.172	.199	.217	.230	.241	.248	.255	.260	.265	.268	.272	.051
12	.105	.146	.172	.190	.202	.212	.220	.226	.231	.235	.239	.242	.039
13	.085	.124	.148	.165	.177	.186	.194	.200	.205	.209	.212	.215	.030
14	.069	.104	.127	.143	.155	.164	.171	.176	.181	.185	.188	.191	.023
15	.056	.087	.109	.124	.135	.143	.150	.156	.160	.164	.167	.170	.018

TABLE [illegible]

Efficiency After k Missions of a System with 4 Part Types

System Sizes, 14-25

$p_1(1) = .[illegible], p_2(1) = .[illegible], p_3(1) = .95, p_4(1) = .97$

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	14	15	16	17	18	19	20	21	22	23	24	25	
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.865	.866	.868	.869	.870	.871	.872	.763
2	.761	.763	.766	.768	.770	.771	.773	.774	.776	.777	.778	.779	.562
3	.679	.681	.684	.686	.688	.690	.692	.693	.695	.696	.697	.698	.444
4	[illegible]	[illegible]	.611	.614	.616	.618	.620	.621	.623	.624	.625	.626	.339
5	.541	.544	.547	.549	.551	.553	.555	.557	.558	.559	.561	.562	.259
6	.484	.487	.489	.492	.494	.496	.497	.499	.500	.502	.503	.504	.198
7	.432	.435	.438	.440	.442	.444	.446	.447	.448	.450	.451	.452	.151
8	.386	.389	.391	.394	.396	.397	.399	.401	.402	.403	.404	.405	.115
9	.345	.347	.350	.352	.354	.356	.357	.359	.360	.361	.362	.363	.088
10	.308	.310	.313	.315	.317	.318	.320	.321	.322	.324	.325	.326	.067
11	.274	.277	.279	.281	.283	.285	.286	.287	.289	.290	.291	.292	.05[illegible]
12	.245	.247	.249	.251	.253	.254	.256	.257	.258	.259	.260	.261	.039
13	.218	.220	.222	.224	.226	.227	.229	.230	.231	.232	.233	.234	.030
14	.194	.196	.198	.200	.201	.203	.204	.205	.207	.208	.209	.209	.023
15	.172	.174	.176	.178	.180	.181	.182	.184	.185	.186	.186	.187	.018

TABLE 7

Efficiency After k Missions of a System with 5 Part Types

System Sizes, 2-13

$p_1(1) = .9$, $p_2(1) = .92$, $p_3(1) = .95$, $p_4(1) = .97$, $p_5(1) = .99$

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	2	3	4	5	6	7	8	9	10	11	12	13	
1	.775	.791	.804	.814	.823	.830	.835	.840	.844	.848	.851	.854	.755
2	.627	.663	.688	.705	.717	.727	.735	.741	.746	.750	.754	.758	.570
3	.521	.569	.598	.617	.631	.641	.650	.657	.663	.667	.672	.675	.430
4	.439	.491	.522	.542	.556	.568	.576	.584	.589	.595	.599	.603	.325
5	.371	.425	.456	.476	.491	.503	.512	.519	.525	.530	.534	.538	.245
6	.314	.367	.398	.419	.434	.445	.454	.461	.467	.472	.477	.481	.185
7	.265	.317	.348	.368	.383	.394	.403	.410	.416	.421	.425	.429	.140
8	.222	.273	.303	.323	.338	.349	.358	.365	.370	.375	.379	.383	.106
9	.185	.235	.264	.284	.298	.309	.317	.324	.329	.334	.338	.342	.080
10	.153	.201	.229	.248	.262	.273	.281	.287	.293	.297	.301	.305	.060
11	.126	.172	.199	.217	.230	.240	.248	.255	.260	.265	.268	.272	.045
12	.103	.146	.172	.189	.202	.212	.219	.226	.231	.235	.239	.242	.034
13	.084	.123	.148	.165	.177	.186	.194	.200	.205	.209	.212	.215	.026
14	.068	.104	.127	.143	.155	.164	.171	.176	.181	.185	.188	.191	.020
15	.055	.087	.109	.124	.135	.143	.150	.156	.160	.164	.167	.170	.015

TABLE 5

Efficiency After k Missions of a System with 5 Part Types

System Sizes, 14-25

$p_1(1)$ = [illegible], $p_2(1)$ = [illegible], $p_3(1)$ = .95, $p_4(1)$ = .97, $p_5(1)$ = .99

No. of Mis. k	Cannibalized System of Size												Uncan. System of Any Size
	14	15	16	17	18	19	20	21	22	23	24	25	
1	.856	.858	.860	.862	.863	.865	.866	.867	.869	.870	.871	.871	.755
2	.761	.763	.766	.768	.770	.771	.773	.774	.776	.777	.778	.779	.570
3	.678	.681	.684	.686	.688	.690	.692	.693	.695	.696	.697	.698	.430
4	.606	.609	.611	.614	.616	.618	.620	.621	.623	.624	.625	.626	.325
5	.541	.544	.547	.549	.551	.553	.555	.557	.558	.559	.561	.562	.245
6	.484	.487	.489	.492	.494	.496	.497	.499	.500	.502	.503	.504	.185
7	.432	.435	.438	.440	.442	.444	.446	.447	.448	.450	.451	.452	.140
8	.386	.389	.391	.394	.396	.397	.399	.401	.402	.403	.404	.405	.106
9	.345	.347	.350	.352	.354	.356	.357	.359	.360	.361	.362	.363	.080
10	.308	.310	.313	.315	.317	.318	.320	.321	.322	.324	.325	.326	.060
11	.274	.277	.279	.281	.283	.285	.286	.287	.289	.290	.291	.292	.045
12	.245	.247	.249	.251	.253	.254	.256	.257	.258	.259	.260	.261	.034
13	.218	.220	.222	.224	.226	.227	.229	.230	.231	.232	.233	.234	.026
14	.194	.196	.198	.200	.201	.203	.204	.205	.207	.208	.209	.209	.020
15	.172	.174	.176	.178	.180	.181	.182	.184	.185	.186	.186	.187	.015

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We consider the performance of a system composed of n machines, each machine being composed of parts (or modules) of r different types arranged in a series. If S_i denotes the number of parts of type i functioning at a given moment, it is assumed that the performance level, Φ, of the overall system at that moment is given by the equation

$$\Phi = \min_{1 \leq i \leq r} S_i \; .$$

A collection of identical structures (ships, aircraft, missile batteries, etc.) exemplifies the kind of system we have in mind, under the simplifying assumption that each part in a given structure is essential to the performance of that structure. Limit theorems

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13. Abstract - continued

are proved which make it possible to evaluate explicitly the asymptotic performance level of the system under three different assumptions:

1. Cannibalization is prohibited.
2. A failed part in a given machine can be replaced by an operating part of the same type taken from another machine. The lifetimes of the parts are mutually independent random variables.
3. A failed part in a given machine can be replaced by an operating part of the same type taken from another machine. Failures of parts may affect the lifetimes of other parts.

It is shown that assumptions (2) and (3) lead to asymptotically equivalent performance levels. The superiority of systems in which cannibalization is practiced over those in which it is prohibited is quantitatively appraised.

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